

Quiz 1 — Theoretical foundations

Questions	Answers
1. What do you do first when studying a time series?	☐ Plot the partial autocorrelation. ☐ Plot the autocovariances. ☐ Check strong stationarity. ☐ None of the above.
2. What is the purpose of the stationarity of a series?	☐ To be sure that our model will still be valid later and will produce consistent forecasts. ☐ To make sure that our residuals are white noise. ☐ To make sure that no deterministic trend can distort our ARMA(p, q) modeling. ☐ Mathematical beauty, with no real interest.
3. What is the definition of weak stationarity of a time series (X_t) ?	☐ For all h and all $k \geq 1$, $(X_1, X_2, \dots, X_k)$ has the same distribution as $(X_{1+h}, X_{2+h}, \dots, X_{k+h})$. ☐ $E[X_t] = 0$, $\text{Var}(X_t) = \sigma^2$, and for all t, s , $\text{Cov}(X_t, X_s) = 0$. ☐ $E[X_t] = 0$, $\text{Var}(X_t) = \sigma^2$, and (X_t) is i.i.d. ☐ $E[X_t]$ does not depend on time, and $\text{Cov}(X_t, X_{t+h})$ does not depend on time for any h .
4. What are the orthogonality conditions of X_t on the Hilbert space $\{1, X_{t-1}, \dots, X_{t-p}\}$?	☐ $E[X_t - \widehat{X}_t] = 0$ and $E[(X_t - \widehat{X}_t)X_{t-j}] = 0$ for $j \in \{1, \dots, p\}$. ☐ $E[(X_t - \widehat{X}_t)X_{t-j}] = 0$ for $j \in \{1, \dots, p\}$. ☐ $\text{Cov}(X_t - \widehat{X}_t; X_k) = 0$ for $k \in \{1, \dots, p\}$. ☐ $\text{Cov}(X_t - \widehat{X}_t; X_k - \widehat{X}_k) = 0$ for $k \in \{1, \dots, p\}$.
5. For $m \in \mathbb{N}^*$, how can you estimate $\text{Corr}(X_t - \widehat{X}_t; X_{t-m} - \widehat{X}_{t-m})$?	☐ Regress X_t on the past observations $X_{t-1}, \dots, X_{t-m+1}$ (coefficients $\beta_1, \dots, \beta_{t-m+1}$): the correlation equals β_{t-m+1} . ☐ Regress X_t on the past observations $X_{t-1}, \dots, X_{t-m+1}$ (coefficients $\beta_1, \dots, \beta_{t-m}$): the correlation equals β_{t-m} . ☐ Regress $X_t - \widehat{X}_t$ on the past residuals $X_{t-1} - \widehat{X}_{t-1}, \dots, X_{t-m+1} - \widehat{X}_{t-m+1}$ (coefficients $\beta_1, \dots, \beta_{t-m+1}$): the correlation equals β_{t-m+1} . ☐ Regress $X_t - \widehat{X}_t$ on the past residuals $X_{t-1} - \widehat{X}_{t-1}, \dots, X_{t-m+1} - \widehat{X}_{t-m}$ (coefficients $\beta_1, \dots, \beta_{t-m}$): the correlation equals β_{t-m+1} .

Questions	Answers
6. How can you check empirically that $X_t - \widehat{X}_t$ is an innovation?	☐ A portmanteau test (Ljung–Box or Box–Pierce). ☐ The question is badly worded. ☐ You cannot: it is a concept we only use in the problem sets.