How to give an economic sense to parameters and equations?

Revision Sheet

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Some disclaimers about the revision sheet. *This revision sheet is provided as an additional support to help you check understanding of the parameters and equations covered in the course. The course and the tutorials remain the only reference. The revision sheet is not necessary, and by all means not sufficient, to study Macroeconomics 1. Nonetheless, it may help you develop intuition and improve your interpretation of macroeconomic models, which can be useful for the exam and beyond. Should you have any question or remark, please reach out to* `clement.montes@ensae.fr` *or* `nina.stizi@ensae.fr`

Objectives of the reminder

1. Explore the assumptions underlying the production function.
2. Review the market clearing conditions.
3. Summarize the optimization problem of each economic agent.
4. Interpret the Euler equation.
5. Distinguish between positive and normative implications.

1 Production

In macroeconomics, it is common to assume that a firm produces its output using capital (K), labor (L), technological efficiency (A) and other production inputs (you have seen the intermediate inputs X, produced by intermediate firms in Römer 1990 (see Chapter 5 and Tutorial 5), the non-renewable energies R in an extension of CKR (see Tutorial 3); but you can think of anything else like natural resources, ...). In class you usually have: $Y = F(K, L)$

1.1 Neo-Classical Production Functions

Most of the time, the production function F is said to be

- essential: there is no output if one factor of production is zero.
- strictly increasing in each of its arguments: meaning that if a factor of production increases, then the firm's production increases too.
- strictly concave in each of its arguments: meaning that the marginal productivity of a factor of production is decreasing; put otherwise, increasing any production input does increase the output, but at a decreasing rate (every increase in a factor has a strictly smaller impact on the final production than the previous increase).
- homogenous of degree 1: meaning that the function has constant returns to scale; put otherwise if I double all my inputs, then my output also doubles.
- satisfying the Inada conditions in each argument: meaning that when a factor of production is scarce (limit of the first derivative close to 0), a small increase in this scarce factor will increase the marginal productivity of this factor to infinity (derivative of F with respect to this factor is $+\infty$), while when it is abundant (limit of the first derivative close to $+\infty$), a small increase in this input will not be significant in the production (marginal productivity of this factor is 0 at the limit).

1.2 CES Production Functions

Most common production functions in Macro are constant elasticity of substitution (CES) production functions. This assumption is, up to a logarithmic transformation, assuming linearity. In CES functions, one factor of production can substitute with another to maintain the same level of production. *Substitution* answers the question "If one input becomes scarce or expensive, how easily can the firm replace it with another input while keeping output constant?".[1] CES functions take the shape:

$$Y = [\alpha_1 \times x_1^\gamma + ... + \alpha_n \times x_n^\gamma]^{\frac{\nu}{\gamma}}$$

with

- x_i's the factors of production (usually $n = 2$ and $x_1 = K$, $x_2 = L$),
- the α_i's are the share of factor x_i in the production[2], namely each factor x_i contributes to $100 \times \alpha_i$ % of the production, and with $\alpha_1 + ... + \alpha_n = 1$[3]
- $\gamma \in (-\infty, 1)$ is the substitution parameter. It governs how quickly marginal productivity falls when one input replaces another, or put otherwise, how fast diminishing returns kick in when you substitute one input for another. It is isomorphic to the elasticity of substitution: $\frac{1}{1-\gamma}$.
  - A high γ means that diminishing returns kick in slowly, so substitution is easy; hence the elasticity of substitution is high.
  - A small γ means that diminishing returns kick in fast, so substitution is hard; hence the elasticity of substitution is low.
  - If $\gamma \in \{0, 1, -\infty\}$, then see respectively special cases Cobb-Douglas, perfect substitutability, and Leontief respectively.
- $\nu \in (0, +\infty)$ is the degree of homogeneity.
  - if $\nu = 1$, homogeneity of degree 1, so constant returns to scale. This is what you have seen in class.
  - if $\nu < 1$, decreasing returns to scale (i.e., if each input of production doubles, the production less than doubles).
  - if $\nu > 1$, increasing returns to scale (i.e., if each input of production doubles, the production more than doubles).

Special Case #1 - $\gamma = 0$ - Cobb-Douglas: $F(x_1, ..., x_n) = x_1^{\alpha_1} ... x_n^{\alpha_n}$ with $\alpha_1 + ... + \alpha_n = 1$

- All the factors of production $x_1, ..., x_n$ are complements.
- The elasticity of substitution is 1 (unit elasticity). Input proportions adjust proportionally to changes in relative prices. Put otherwise, a change in relative prices leads to a proportional change in input ratios.

Special Case #2 - $\gamma = 1$ - Perfect substitutability: $F(x_1, ..., x_n) = \alpha_1 x_1 ... \alpha_n x_n$ with $\alpha_1 + ... + \alpha_n = 1$

- All the factors of production $x_1, ..., x_n$ are perfect substitutes.
- The elasticity of substitution is $+\infty$. Input proportions adjust infinitely strongly to relative price changes. Put otherwise, even an arbitrarily small change in relative prices leads to a corner solution, with the firm using only the cheaper input.

Special Case #3 - $\gamma = -\infty$ - Leontief $F(x_1, ..., x_n) = \min(\frac{x_1}{\alpha_1}, ..., \frac{x_n}{\alpha_n})$ with $\alpha_1 + ... + \alpha_n = 1$

- All the factors of production $x_1, ..., x_n$ are perfect complements: whatever the quantity of one input, the production is limited and equal to the scarcest of them, relative to their contribution to the production. Put otherwise, production can only happen if all the factors are used.
- The elasticity of substitution is 0. It is impossible to substitute. Whatever the change in relative prices, input proportions do not adjust at all. All inputs are required in fixed proportions to produce output.

[1] The elasticity of substitution is a more complex notion that measures how responsive the input ratio K/L is to changes in the marginal rate of technical substitution (MRTS). The MRTS is the ratio of the marginal productivities (of K and L respectively). Graphically, the MRTS is interpreted as the slope of the isoquants. The MRTS shows the rate at which one input (K or L) may be substituted for another while maintaining the same level of output. For CES functions, the elasticity of substitution is constant and depends only on the substitution parameter (hereafter, γ), meaning that the MRTS doesn't need to be computed.

[2] This interpretation implicitly assumes that input markets are in pure and perfect competition. This assumption allows to test the fit of the models.

[3] This assumption is not equivalent to constant returns to scale, it comes from normalization. The returns are dictated by ν.

2 Markets and Market Clearing Conditions

In the long-run growth models, it is assumed, unless said otherwise (cf. Römer 1990) that markets are:

a) in pure and perfect competition (i.e. large number of buyers and sellers, homogeneous product, free entry and exit, perfect information, perfect mobility of factors of production):

- *Implication 1*: agents are price takers not price setters.
- *Implication 2*: factors of production are remunerated to their marginal productivity. Put otherwise, the cost of a factor of production is equal to the price of this factor, which is its marginal productivity. For example: the cost of labor, that is, the real wage, is equal to the marginal productivity of labor.
- *Implication 3*: market suppliers (firms in the goods market, for instance) make zero profit in the long run.
- *Implication 4*: absent externalities, provided markets are complete, and the model encompasses a finite number of agents, the 1^{st} welfare theorem applies (market equilibrium is Pareto-optimal).

b) at equilibrium:

- Implication: the prices (taken by the agents) should clear the market.

Market	Supply	Demand	Market Clearing Condition and its Consequence(s)
Final Goods	Firms	Households	Firms produce final goods that are used by households for consumption, C_t, and saving, S_t, and by government for public spending G_t. Aggregate demand in a closed economy is $$C_t + S_t + G_t$$ where $G_t \neq 0$ only when public spending and transfers are considered in the model (Chapters 6-7 and Tutorial 6). In that case S_t can be interpreted as an aggregate of public and private savings. Final Good Market Clearing Condition: Aggregate supply by firms Y_t = Aggregate demand from households i.e. $Y_t = C_t + S_t + G_t$ Consequence: Considering the special case of national income accounting identity where investment = savings (IS equation), we derive the Law of accumulation of capital: Capital accumulates through investments and depreciates at rate δ. $$\dot{K}_t \equiv I_t - \delta K_t = (Y_t - C_t) - \delta K_t$$
Labor	Households	Firms	Each individual supplies one unit of labor to the firm. Hence, the aggregate labor supply is equal to the population size, L_t, and the aggregate labor demand is the sum of individual labor demands N_{it} across all firms i in the set of final-good producers. Labor Market Clearing Condition: Aggregate labor supply = Aggregate labor demand i.e., $L_t = N_t$ with $N_t = \sum_{i \in I} N_{it}$

Bonds	Households	Households	Bonds are riskless one-period assets. They are traded only among households at a price equal to the real interest rate, r_t. Since bonds are traded only among households and no agent outside the household sector issues them, bonds are in zero net supply in equilibrium. Put otherwise, every bond held by a household is issued by another household. Therefore, at the equilibrium, aggregate bond holdings sum to zero. Bonds Market Clearing Condition: $$\begin{aligned} &\text{Aggregate demand for bonds} = \text{Aggregate supply of bonds} \\ \text{i.e.} \quad &\text{Aggregate Bond Holding} = \text{Total bonds held - Total bonds issued} \\ &= 0 \end{aligned}$$ Consequence: Households' aggregate financial wealth equals the aggregate capital stock. $$\underbrace{\text{Aggregate Financial Wealth}}_{B_t} \equiv \underbrace{\text{Aggregate Bond Holdings}}_{0} + \underbrace{\text{Capital Stock}}_{K_t}$$ $$B_t = K_t$$
Loans	Households	Firms	Firms demand loans to finance accumulation. Households lend to firms. The market clears when loan demand from firms equals the loan supply by households. Loans Market Clearing Condition: $$\text{Aggregate demand for loans} = \text{Aggregate supply of loans}$$ Consequence: $$r_t = z_t - \delta \iff r_t = \frac{\partial F(K,L)}{\partial K} - \delta$$ This result is obtained either through the borrowers' (i.e., the firms') perspective or through the lenders' (i.e., the households') perspective. • *Borrowers' pespective:* Firms borrow until $$\underbrace{\text{Marginal benefit of capital}}_{\underbrace{z_t}_{=\frac{\partial F(K,L)}{\partial K}} - \delta} = \underbrace{\text{Marginal cost of borrowing capital}}_{r_t}$$ The marginal benefit of capital equals the marginal product of capital ($z_t = \frac{\partial F(K,L)}{\partial K}$) net of depreciation. The marginal cost of borrowing capital is the price of a loan, that is the real interest rate r_t. • *Lenders' perspective*: Households are indifferent between lending to firms (via loans to finance capital) and lending to other households (via riskless one-period assets) if the remuneration of the first, that is marginal product of capital net of depreciation $z_t - \delta$ with $z_t = \frac{\partial F(K,L)}{\partial K}$, is the remuneration of the second, that is the real interest rate r_t. If one asset yields a higher return, then all households will engage in only one market, leaving the other market incomplete.

Römer (1990) model (see Chapter 5) introduces an intermediate goods market that is not purely and perfectly competitive, but instead operates in monopolistic competition[4]. In the presence of a monopolistically competitive market,

- the producer is not price taker but price setter.
- prices are no longer given by the marginal productivity but rather by a function of the marginal productivity and the markup, which allow producers to earn positive profits.
- the level of production is suboptimal (below competitive market equilibrium) because prices are higher, so not welfare-maximizing.

3 Maximization Objectives for the Economic Agents

In neoclassical economics, agents are assumed to be rational (i.e., they make choices in order to maximize their objective function). This section briefly reviews the maximization objectives of each agents seen in the course and tutorials material.

	Household	**Firms**	**Social Planner** (Benevolant Omnipotent Omniscent Planner, or BOOP)
Objective	Individual intertemporal utility maximization	Individual instantaneous profit maximization	Intertemporal social utility maximization
Variables of Decision	- Consumption path $(c_t)_{t\geq 0}$ - Asset stocks path $(b_t)_{t\geq 0}$ (Labor is supplied inelastically)	- Quantity of each inputs (usually K and L), - Price if monopolistic competition	- Consumption path $(c_t)_{t\geq 0}$ - Capital path $(k_t)_{t\geq 0}$
Economic mechanisms	- Trade-off saving-consuming - Trade-off consuming today vs. tomorrow	Trade-off investing to increase stock of capital vs. producing	
Time Horizon	Intertemporal: at a given date t_0, she chooses a path over an infinite horizon (households are infinitely lived).	Instantaneous: the choices of the factors of production and production is made at every period.	Intertemporal
Choices	**Consumption path that maximizes utility**. From consumption path, one can deduce saving path and, at equilibrium, the investment path, which serves the firms.	(i) **Factors of production** (capital and labor most commonly, technological progress $A_t = \frac{K_t}{L_t}$ in the learning-by-doing Römer 1986 model (Chapter 4), intermediate inputs in the product variety Römer 1990 model (Chapter 5) or final good used in the production of the intermediate good) and (ii) Only for the intermediate firm that produces variety j in Römer 1990 (Chapter 5 and Tutorial 5), **individual production** X_{jt} (but not for the final firm!). **that maximize profit at every period**.	(i) **Initial consumption level c_0 (or equivalently γ_0)** that sets the economy on the balanced growth path/saddle path[5] (ii) **Aggregate production**, so as to allow households to consume optimally. In the BOOP program, there is no market: she computes the optimal quantities produced and consumed under the constraint of resources.

[4]The term competition is coined by the fact that the inventors produce differentiated goods (no longer homogeneous), and thus still have to compete with one another through substitution across inputs.

[5]The planner chooses the paths $(c_t)_{t\geq 0}$ and $(k_t)_{t\geq 0}$ in order to maximize the utility of the households. While the initial capital stock k_0 is given, the initial consumption level c_0 is initially not. Instead, c_0 is a control variable chosen optimally by the planner. Going further, it is wrong to say that the planner adjusts consumption conditional on c_0. Instead one can say that the planner chooses the entire consumption path $(c_t)_{t\geq 0}$ that maximizes the households' utility, conditional on the initial stock of capital k_0.
How does the planner chooses c_0? For any level $k_0 \in (0; +\infty)$, there is only one consumption level c_0 that respects (a) the Euler

At optimum	(i) **Instantaneous budget constraint.** Earnings from labor and financial assets diminished by consumption determines the evolution of the stock of assets each period. (ii) **Non-negativity of consumption.** (iii) **Solvency condition.** The discounted value at date 0 of intertemporal debt (or financial assets) is greater than zero. It ensures that agents do not accumulate debt forever without repayment. This rules out Ponzi schemes. It allows to obtain the intertemporal budget constraint. and (iv) **Transversality condition.** The discounted value of debt converges to zero in the limit, ensuring that agents do not leave unused wealth. [6].	(i) **Capital dynamics** (ii) **Demand-driven production.** Production matches the demand, because of market equilibrium. The level of production needed determines the **optimal bundle of inputs**. The price of the good is given when its market is perfectly competitive. Price of final good is even normalized to one thanks to Walras law in all frameworks studied.	(i) **Resource constraint** coincides with the equation of the equilibrium on the final good market. (ii) **Aggregate production.** It is such that consumption of the representative household is maximized

4 Euler Equation

$$\frac{\dot{c}_t}{c_t} = \sigma(c_t)(r_t - \rho) \overset{CRRA}{=} \frac{1}{\theta}(r_t - \rho) \tag{1}$$

The Euler equation describes how the growth rate of consumption responds to changes in the interest rate, the time-preference parameter, and the curvature of the utility function.
Hereafter, each component of the Euler equation is explained.

- **Consumption growth rat $\frac{\dot{c}_t}{c_t}$.** Let $c_t \in \mathbb{R}^+$ be the consumption per capita at date t, so $\frac{\dot{c}_t}{c_t} \in \mathbb{R}$ is the growth rate of consumption (in percentage). If $\frac{\dot{c}_t}{c_t} > 0$ (resp. < 0), consumption in the future is larger (resp. smaller) than consumption today, so consumption growth (resp. declines) at rate $\frac{\dot{c}_t}{c_t}$ %.
- **Real interest rate r_t.** $r_t \in (0, +\infty)$ is the remuneration in terms of goods[7] of households' supply of credits. Put otherwise, if a household saves one unit of good in period t, this household receives $(1 + r_t dt)$ unit of good at date $t + dt$; that is her unit of good lent in date t plus an extra to compensate for the cost of not having consumed this unit at date t.
  - If r_t is low, the extra she could have is very small, hence she tends to save less today and present consumption goes up (at the expense of future consumption).

equation, (b) the feasibility condition, and (c) the transversality constraint, that is one unique c_0 that puts the economy on the saddle path. The BOOP chooses this unique value as an initial level of consumption.

[6]While the transversality condition is an **optimal** behavioral restriction that selects economically meaningful solutions, the solvency condition is about feasibility of repayment. The transversality condition implies that the intertemporal budget constraint is binding, i.e., that we have an equality between the intertemporal flow of actualized consumption and the sum of the intertemporal flow of actualized labor income and initial stock of financial wealth. The solvency condition implies that intertemporal flow of actualized consumption is bounded upward by the sum of the intertemporal flow of actualized labor income and initial stock of financial assets. While the solvency condition always holds, the transversality condition only holds at optimum.

[7]In Macroeconomics 1, we will always look at values in *real* terms. Unlike *nominal* terms, real terms allow to study the evolution of the production in volume and get rid of the price effect (inflation). It allows to (a) have only one final good that serves all purposes and understand the dynamics of production growth in that simple framework, and (b) avoid considering price fluctuations. If you want to dig deeper or go a step further, you could be interested in monetary policy questions or asset pricing models that consider them.

- If r_t is high, saving will bring her a jackpot at a later, so she saves more, decreasing therefore her current consumption to the benefits of future consumption.

Put differently, an increase of the interest rate at date t, makes it more advantageous to save (and hence consume less) at date t in order to consume more at a later date $t + dt$. On the contrary, when interest rate decreases, the incentive to save declines, hence the present consumption rises at the expense of future consumption, henceforth the consumption growth rate declines.

- **Time preference parameter ρ.** $\rho \in \mathbb{R}$ is also called the parameter of preference for the present. It measures how much a household values receiving consumption earlier rather than later.
  - If $\rho \geq 0$, delaying consumption is costly, which is the standard assumption.
  - If ρ is close to 0, postponing consumption is relatively painless.

  $r_t - \rho$ measures the propensity to save. Indeed, the difference of the endogenous variable and the parameter measures the will of the household to save. If $\rho > r_t$, the remuneration she gets out of saving does no compensate the cost of delaying her consumption, so she does not save. If $\rho < r_t$, saving brings her more than it costs her, so she does save. If $\rho = r_t$, she is indifferent between saving and consuming today. By assumption, we consider that in this case the household saves (in order to support investment).

- **Risk aversion of the household θ.** In the special form of CRRA utility[8] seen in class, $\theta \in (0, +\infty)$ measures the curvature of the utility function.
  - If θ is high, the household is very risk-averse and would rather consume today than risk it on tomorrow. The household does not want to smooth its consumption across time.
  - If θ is low, it is less costly to postpone consumption and the household is thus more willing to smooth consumption.

  Intertemporal elasticity of substitution (IES) $\frac{1}{\theta}$. Still in the CRRA case, it measures how much consumption growth rate responds to a change in the propensity to save.
  - If the IES is large (i.e., θ close to 0), it is not costly to postpone consumption to a future period, hence the household is more prone to shift her consumption across time, smoothing it at the same time, therefore the consumption growth rate reacts strongly to a change in the propensity to save.
  - If the IES is exactly equal to 1 (i.e., $\theta = 1$), a change in the propensity to save translates one-for-one into changes in the consumption growth rate.
  - If the IES is small (i.e., θ is larger than 1), small changes in the consumption smoothing is costly, hence households change very little their saving/consumption behavior. Therefore the consumption growth rate does not evolve much when propensity to save changes.

5 Questions Addressed by Macroeconomic Models

Positive v/s normative implications. When working with a macroeconomic model, one can draw implications that are either positive or normative.

- A **Positive implication** is a descriptive consequence of the model. It answers questions such as: what happens if this parameter increases? How does one variable respond to changes in another? It is purely factual and analytical: they describe what the model predicts, without value judgments.
- A **Normative implication**, by contrast, involves a judgment. It tackles questions such as: What should be done? If outcomes are unfair/suboptimal, how could they be corrected? Normative implications typically take the form of policy recommendations for policymakers or a social planner, such as taxes, subsidies, caps, quotas, or redistribution schemes. The normative implications depend on the intention of the social planner (e.g., Does she seek to maximize households' long-term or short-term consumption? Does she want to favor production, capital accumulation, human capital accumulation, or other factors?, etc.).

[8] In class, applications focused on the Constant Relative Risk Aversion (CRRA) utility that takes thes following form

$$u(c_t) = \begin{cases} \frac{c_t^{1-\theta} - 1}{1-\theta} & \text{if } \theta \neq 1 \\ \ln(c_t) & \text{if } \theta = 1 \end{cases}$$

6 Positive Implications and Stylized Facts

Stylized facts. Macroeconomic models aim to explain broad empirical regularities observed in real-world data. These observations are called stylized facts. In class, you have seen the 6 Kaldor stylized facts (see Chapter 4, slides 32/53). We saw that a discrepancy between the calibration of a parameter of a model and those stylized facts could help build a more comprehensive framework better depicting reality (Tutorial 1).

Conditional convergence, but no absolute convergence. One well-known stylized facts is conditional convergence. It states that economies sharing the same characteristics (production function, preference parameters, dynamics) tend to converge to similar growth rate,regardless of their initial conditions. However, there is no sign of absolute convergence in the data (Chapter 1, slides 47-48/62). Solow-Swan model (Chapter 1) and Cass-Koopmans-Ramsey model (Chapter 2) generate this conditional convergence. However, Römer's models (Chapters 4 and 5) fail to account for it. Nonetheless, the latter better captures Kaldor's 6th-stylized fact *"the growth rate of per-capita output varies across countries when the preference parameters vary across countries"*. There is no perfect model, one needs to carefully choose the more adapted to his/her question.

No Ricardian equivalence. Another long-studied fact is the equivalence in whether economic policies impact the economy depending on their source of financing. It is not robust empirically, and we even demonstrated that Cass-Koopmans-Ramsey actually shows the reverse (see Tutorial 6). Even though these tests are positive descriptions of the models, they are still helpful in helping to design policies.

$\Longrightarrow$ Bottom-line: A macroeconomic model is an attempt to replicate the dynamics observed in reality using a simplified set of economic variables, with the goal of improving predictions. Its purpose is to make explicit mechanisms that are present in the data but not immediately disentangled. The question of "how to react" belongs to the normative side of the analysis and is addressed in the next section.

7 Normative Implications and Policy Instruments

Common setting for normative analysis in Macroeconomics 1. On at least one of the markets studied in the model, one side of the market generates an externality, whether it is positive (like knowledge spillovers in Römer models, Chapter 4, Tutorial 4) or negative (like pollution in the DICE Nordhaus model seen in Chapter 3, or monopolistic competition seen in Chapter 5, Tutorial 5). This implies that the first welfare theorem cannot apply (i.e., you expect a mismatch between the social optimum and the private decentralized equilibrium). Then, the problem is usually structured as so:

1. Start by computing the decentralized equilibrium: find equilibrium conditions and derive the optimal choices of each agent.
2. Then, compute the social optimum (i.e. the BOOP program): it will help understand how does the equilibrium found above modifies when accounting for the externality spillovers.
3. Compare the two. In case of a positive (resp. negative) externality, because private agents are not remunerated for the benefits (resp. do not account for the costs) they generate, they do not integrate the externality in their maximization problem (i.e. they do not internalize it) so they produce less (resp. more) than optimally desired.
4. Study what tax/subsidy/quota could the planner implement in order to make the decentralized equilibrium (with the policy instrument chose) coincides with the social optimum.

Policy instruments. A common policy instrument is to introduce a tax or a subsidy $\tau \in \mathbb{R}$ that will affect directly one side of the market in order to correct for market inefficiency/market distortion in presence of an externality. Below are two common examples that could help you understand.

Policy instrument example #1: tax/subsidy on financial income: Suppose the planner wants to tax/subsidize private financial revenue in order to slow down/stimulate savings (and consequently investments). To do so, she introduces a tax/subsidy on households' financial revenues. Letting $\tau \in \mathbb{R}$ denote the policy instrument. The return on savings is then modified from r_t to $(1-\tau) \times r_t$. For one bond saved, the household, who should receive $1 + r_t$ unit at the next period absent any policy (i.e., when $\tau = 0$), will instead receive $1 + (1-\tau) \times r_t$ unit.

- If $\tau < 0$, the remuneration of financial asset is higher than before, so households have an incentive to save more. They are subsidized to save. The Euler equation indicates that consumption declines while saving increase. At the equilibrium it generates more investment, hence more production.

- If $\tau > 0$, the remuneration is lower, so the policy instrument τ is a tax. By a similar reasoning production decreases.

It is important to pay close attention to the sign convention: had the return been written as $(1 + \tau) \times r_t$ the interpretation of the sign of τ would be reversed.

Policy instrument example #2: correcting monopolistic distortion: In the Römer (1990) growth-with-variety model (Chapter 5), intermediate-good producers operate in monopolistic competition, each producing a specific variety j. Therefore, inventor j produces sub-optimally (higher price, less quantities than Pareto-efficient according to the First Welfare Theorem). The BOOP has several options to correct this distortion:

- Subsidize the purchase of intermediate goods by final firms. It reduces the cost of intermediate inputs while still allowing innovation incentive, to increase final production. She can finance the subsidy with a lump-sum-tax that creates no distortion on the households' side.
- Subsidize the production of final goods. It increases the demand of final good producers for inputs (i.e. intermediate goods) so that intermediate-producers are forced to supply more, reaching the Pareto-efficient quantity. She can finance the subsidy with a flat-tax that creates no distortion on the households' side.

Government budget. The budget of the government must be balanced. The government needs to leverage taxes to finance subsidies. By assumption, there is no possibility for a government to get indebted and reimburse later. For every spending done, an income needs to be raised. It is then important to study the reaction of agents to that new tax inside of the same model. It may be that the reactions of one agent partially offset the correction of the original externality: the design of instruments need to account for this as well. Below are two examples we studied during Tutorial 4 that could help you understand.

Funding the policies example #1: tax/subsidy on labor income: Labor income is a natural source of government revenue. Because the household supply labor inelastically, changing the net wage she obtain will not change her labor supply. This result is in line with the assumption that household utility depends only on consumption.

Funding the policies example #2: lump sum tax on households: The externalities studied in tutorials were all impacting the firms producing. The counterbalancing forces to pay for the externality may then come from the household. Adding a tax that is not *ad valorem* (i.e., not impacting any of its price in a $(1 - \tau)$ fashion) to her budget constraint will not modify her behavior: this is the definition of a lump-sum tax.